Reconsidering the home market effect in a model with costly foreign investment

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Abstract. The aim of this paper is reconsidering the home market effect (HME) in a model with costly foreign investment. We extend the two-sector, two-factor model by Takatsuka and Zeng (2012) to allow for the Samuelson's iceberg transport costs in international capital movement. Using a model with perfectly integrated capital market, Takatsuka and Zeng (2012) shows the appearance of the HME. By introducing an assumption of costly foreign investment, our model shows that an existence of large transport cost in capital movement hinders the appearance of the HME. The effect of a reduction in the transport cost in capital movement is generally unclear, because it affects (i) the *relative price advantage*, and (ii) the *relative market size* between two countries in opposite directions. However, as long as the market for industrial (differentiated) goods is already integrated sufficiently, a reduction of the transport cost in capital movement clearly contributes to increases the share of firms in the small-population (and less-wealthier) country and promotes industrialization in that country.

Keywords: home market effect; agglomeration; transport costs; costly foreign investment; international capital movement.

1 Introduction

Does a wealthier country always attract more firms (and more investment) than its share of population in the whole economy? If it does, we state that "the home market effect (HME) occurs" in the literature on the New Trade Theory. The aim of this paper is reconsidering the HME in a model with costly foreign investment. We extend the two-sector, two-factor model by Takatsuka and Zeng (2012) to allow for the Samuelson's iceberg transport costs in international capital movement.[*1] This type of transport costs are modeled by Zeng (2016)

[*1] Takatsuka and Zeng (2012) is an extended model of the footloose capital model of Martin and Rogers (1995).

which assumes only one sector in the economy.

In the literature on the HME, many of the earliest studies are based on one-factor (i.e., labor) models with an *identical* wage rate (see Helpman and Krugman (1985, Section 10.4), among all). So in those papers *a country with larger population* means one with larger market size (i.e., *wealthier in national income*). In contrast, Takatsuka and Zeng (2012) examines the HME in a two-factor (i.e., labor and capital) model with *unequal* wage rates but *identical* capital rents between two (small and large) countries that are only different in population each other. In their model a country importing the agricultural good (with positive transport costs) always becomes a higher-wage country; and they show that the large-population country never be an exporting country of agricultural good and capital. It implies that their model inherits the property that a country with *larger population* is one with *wealthier in national income*.

By introducing transport costs in international capital movements in the model of Takatsuka and Zeng (2012), our model may generate *unequal* capital rents in each country as well as *unequal* wage rates. If the residents of the small-population country earn higher wage and capital rents, the small-population country may exceeds the other (large-population) country in national income. However, the reason behind the statement above by Takatsuka and Zeng (2012) also holds in our model, so that the large-population country never be an exporting country of agricultural good and capital. Thus, import of agricultural good by the large-population country generates a higher wage in the country, and import of capital by the country generates a higher capital rents in the country, respectively. It will be explained later, foreign investment is assumed to be more costly than domestic one. So a higher capital rent in the large-population country benefits the residents there more largely than the residents of the small-population country. Therefore, it also holds in our model that a country with *larger population* is one with *wealthier in national income*. Therefore, our strategy of the analysis is discovering how the transport costs in international capital movement hinder the incentives of investors living in two-country economy, where the same situation with preceding studies holds.

The remainder of this paper is organized as follows. In section 2, we present a extended model of Takatsuka and Zeng (2012), by introducing costly foreign investment. In section 3 our main analysis is explained in an equilibrium with both types of goods are traded simultaneously. In section 4 we show an equilibrium analysis when the homogeneous good is not traded internationally because of large transport cost of the good. Section 5 contains conclusions.

2 The model

Consider a world economy consisting of two countries, 1 and 2. The countries have the same physical and geographical constraints, except for population size. The population of Country 1, $L_1 = \theta L$, is larger than that of Country 2, $L_2 = (1-\theta)L$, where L is a constant world population and $\theta \in (1/2, 1)$ is the share of Country 1 in it. Each individual owns one unit of labor and κ units of capital, and supplies them inelastically. Each country consists of two sectors, agricultural and manufacturing. Assume that the consumption share of agricultural good is large enough so that both countries always produce the good.

Labor is immobile, while capital and the two types of goods are mobile across countries with Samuelson's iceberg transport costs. In the manufacturing sector *bilateral* intra-industry trade can occur, while international trades of agricultural good and capital are *monolateral*. The patterns of trades are important in our model to determine which country has a larger national income. Since our model generates unequal wage rates and capital rents between two countries, it is noteworthy that whether if a larger-population country always be a wealthier country in national income.[*2] As explained above, in our model the country with larger population, Country 1, always be the wealthier one in national income.

2.1 Preferences

All consumers have identical preferences. The utility function of a representative consumer in Country i is

$$U_i = M_i^{\mu} A_i^{1-\mu}, \quad \mu \in (0, 1), \quad i \in \{1, 2\}, \tag{1}$$

where M_i is consumption of the composite of differentiated manufactured goods (each called a variety, henceforth), A_i is consumption of an agricultural good (Good A, henceforth), and μ is a constant parameter. The CES (constant elasticity of substitution) sub-utility function is defined by

$$M_i = \left[\int_0^{n_i} m_{ii}(v)^{\rho} dv + \int_0^{n_j} m_{ji}(v)^{\rho} dv\right]^{\frac{1}{\rho}}, \quad i, j \in \{1, 2\}, \quad i \neq j, \tag{2}$$

where $\rho \in (0, 1)$ is a constant parameter which determines the elasticity of substitution between different varieties, $\sigma = 1/(1-\rho) > 1$; n_i is the mass of varieties produced in Country i; and $m_{ji}(v)$ is demand for a variety produced in Country j and consumed in Country i (so $m_{ii}(v)$ stands for demand for domestic products). Since each variety is produced by a

[*2] Takatsuka and Zeng (2012) shows that the larger country never be an exporting country of agricultural good and capital. Borrowing their setups, our model shows the same pattern with their study: whenever international movements of agricultural good and capital occur, they move from the smaller country to the larger country. So we can focus on a few trade patterns without checking whether if all possible trade patterns can be realized in equilibrium.

unique firm, n_i is also the mass of firms locating in Country i. The price index for M_i can be obtained as

$$P_i=\left[\int_0^{n_i} p_{ii}(v)^{1-\sigma}dv+\int_0^{n_j} p_{ji}(v)^{1-\sigma}dv\right]^{\frac{1}{1-\sigma}},\quad i,\ j\in\{1,2\},\quad i\neq j, \tag{3}$$

where $p_{ii}(v)$ is price of a domestic variety; and $p_{ji}(v)$ is price of a variety produced in Country j and consumed in Country i, which includes international transport costs.

Let Y_i be the national income and p_i^A be the market price (consumer price) of Good A in Country i. The Marshallian demand functions are derived as follows.

$$\begin{aligned} &A_i=\frac{(1-\mu)Y_i}{p_i^A},\quad M_i=\frac{\mu Y_i}{P_i},\\ &m_{ii}=\frac{p_{ii}^{-\sigma}}{p_i^{1-\sigma}}\mu Y_i,\quad m_{ji}=\frac{p_{ji}^{-\sigma}}{p_i^{1-\sigma}}\mu Y_i,\quad i,\ j\in\{1,2\},\quad i\neq j, \end{aligned} \tag{4}$$

where the demands for all varieties are symmetric, so we omit v, henceforth.

2.2 The agricultural sector

The agricultural sector produces a homogeneous good, named A. It is produced under perfect competition and constant returns to scale, employing labor only. One unit of labor is required to produce one unit of the good, so the profit-maximizing price of the good equals the domestic wage rate in each country. We normalize the wage rate in Country 2 as $w_2=1$, and denote that in Country 1 by $w_1=w$. The Samuelson's iceberg transport cost is assumed whenever goods are shipped to the other country. A parameter $t\geq 1$ represents the cost of transporting Good A; that is, t units of Good A have to be sent for one unit to be delivered to the other country. Then, the producer prices (mill prices) of Good A, a_i, and the consumer prices of the good produced in Country j and consumed in Country i, a_{ji}, are given by

$$a_1=a_{11}=w,\quad a_{12}=tw,\quad a_2=a_{22}=1,\quad a_{21}=t. \tag{5}$$

In principle, there are three possibilities for the trade patterns of Good A:

1. No trade of Good A. Then, the prices meet $p_1^A=w$ and $p_2^A=1$.
2. Country 1 imports Good A. Then, $p_1^A=w=t$ and $p_2^A=1$.
3. Country 2 imports Good A. Then, $p_1^A=w$ and $p_2^A=1=tw$.

However, as mentioned above, the last case cannot emerge in equilibrium, shown by Takatsuka and Zeng (2012). So we focus on the first two cases, where the equilibrium wage differential is not less than one: $w = w_1/w_2 \geq 1$.

2.3 The manufacturing sector

The manufacturing sector produces differentiated varieties. Each of them is produced by monopolistic competitive firms with an increasing returns to scale technology, employing both labor and capital. To model the Samuelson's transport cost in trading varieties, a parameter $\tau \in (1, \infty)$ is defined here; that is, τ units of a variety have to be sent by the producers for one unit to be delivered to the foreign consumers. Let p_i be a producer price of a variety in Country i; p_{ji} be a consumer price of a variety produced in Country j and consumed in Country i, including transport costs; and p_{ii} be a consumer price of a variety produced and consumed in a same country. It holds that

$$p_{11} = p_1, \quad p_{12} = \tau p_1, \quad p_{22} = p_2, \quad p_{21} = \tau p_2.$$

The production technologies of all firms are same, then a symmetric equilibrium occurs. Assume that one unit of capital is employed as a fixed input, and ρ units of labor are employed as constant marginal inputs.[*3] Then, the profit of a firm producing a variety is

$$\begin{aligned} \pi_i &= p_{ii} m_{ii} + p_{ij} m_{ij} - \rho w_i (m_{ii} + \tau m_{ij}) - r_i \\ &= p_i (m_{ii} + \tau m_{ij}) - \rho w_i (m_{ii} + \tau m_{ij}) - r_i \\ &= (p_i - \rho w_i) q_i - r_i, \quad i \in \{1, 2\}, \end{aligned} \tag{6}$$

where q_i denotes amounts of production of the firm:

$$q_i = m_{ii} + \tau m_{ij}, \quad i, j \in \{1, 2\}, \quad i \neq j. \tag{7}$$

Maximizing the profit, we have the following prices and price indices:

$$\begin{aligned} &p_{11} = w, \quad p_{22} = 1, \quad p_{12} = \tau w, \quad p_{21} = \tau, \\ &P_1 = (w^{1-\sigma} k_1 + \phi k_2)^{\frac{1}{1-\sigma}}, \\ &P_2 = (\phi w^{1-\sigma} k_1 + k_2)^{\frac{1}{1-\sigma}}. \end{aligned} \tag{8}$$

where $\phi \equiv \tau^{1-\sigma} \in [0, 1)$ is called trade freeness, and k_i denotes the amount of *effective* capital employed in Country i; note that $k_i = n_i$ holds throughout the paper. As defined later,

[*3] The assumption on the amounts of the marginal factor inputs is useful to simplify our calculation.

because foreign investments inevitably waste a certain amount of the invested capital, we must distinguish between the wasted capital and effective capital.

Because of the CES framework, the income distribution rates in the manufacturing sector are fixed: $1/\sigma$ is the share of the firm's revenue paid for the fixed input (i.e., capital) and $(\sigma-1)/\sigma$ is the share of it paid for the variable input (i.e., labor). Thus, it holds that

$$(\sigma-1)k_i r_i = L_i^M w_i, \quad i \in \{1, 2\}. \tag{9}$$

where r_i is capital rent in Country i, and L_i^M is total labor employment in the manufacturing sector of Country i: $L_i^M = \rho q_i k_i$. Thus, (9) can be solved to derive the equilibrium production of a firm as

$$q_i = \frac{\sigma r_i}{w_i}, \quad i \in \{1, 2\}, \tag{10}$$

which can be also derived from free-entry (zero-profit) condition.

The capital rents, r_1 and r_2, do not equalize, since we assume investments in a foreign country is more costly than that in a domestic country. Following Zeng (2016), we model the costly foreign investments as same as Samuelson's iceberg transport cost: that is, if one unit of capital is invested abroad, only $\gamma \in (0, 1]$ units of it can arrive to the destination, while $(1-\gamma)$ units of it 'melt' along the way. We call the γ units as effective capital that can make returns; and the $(1-\gamma)$ units as wasted capital (or transportation loss) because it does not make any returns. The constant discounted ratio γ represents a degree of capital freeness: $\gamma=1$ means perfect capital mobility, and any reduction in γ stands for a stronger barrier in foreign investments. Let k_{ji} denote the quantity of capital held by residents in Country j but shipped to Country i (including transportation loss). Total effective capital employed in Country i is

$$k_i = k_{ii} + \gamma k_{ji}, \quad i, j \in \{1, 2\}, \quad i \neq j. \tag{11}$$

Inequality $k_{21} > 0$ holds iff $r_2 \leq \gamma r_1$ while inequality $k_{12} > 0$ holds iff $r_1 \leq \gamma r_2$. Clearly, it is impossible for both countries to simultaneously invest in foreign countries unless $\gamma=1$. Note that the fixed capital endowment in the whole economy, κL, must be distributed into two countries' effective capital and transportation loss:

$$\begin{aligned} \kappa L &= k_1 + k_2 + (1-\gamma)k_{12} + (1-\gamma)k_{21} \\ &= k_1 + k_2 + (1-\gamma)k_{21}, \end{aligned} \tag{12}$$

where $k_{12}=0$ always holds, since we do not consider any capital movement from Country 1 to Country 2. In this paper we consider the following three cases in international capital movement:

1. Domestic investment only; with $\gamma r_1 < r_2 < r_1$ and $k_{12}=k_{21}=0$ hold.
2. Some capital moves from Country 2 to Country 1; with $\gamma r_1 = r_2$, $k_{12}=0$, and $0 < k_{21} < \kappa(1-\theta)L$ hold.
3. Full agglomeration in Country 1; with $\gamma r_1 > r_2$, $k_{12}=k_{22}=k_2=0$, $k_{21}=\kappa(1-\theta)L$, and $k_1=(\theta+\gamma-\theta\gamma)\kappa L$ hold.

In the second case above, the total amount of effective capital (i.e., the total number of firms) in the whole economy can vary, since it is affected by changes in the amount of foreign investment. To understand the meaning, it is useful to transform (12) into

$$(\theta+\gamma-\theta\gamma)\kappa L = k_1 + \gamma k_2, \quad \text{iff}\, \gamma r_1 = r_2, \tag{13}$$

where a negative relation between k_1 and k_2 is shown. Therefore, if a unit of capital is shifted from Country 2 to Country 1, $(1-\gamma)$ units of the mass of firms in the whole economy decrease. Clearly, when the case of full agglomeration in Country 1 occurs, the total number of firms in the whole takes the minimum value: $k_1+k_2 = (\theta+\gamma-\theta\gamma)\kappa L$ (with $k_2=0$); while when the case of domestic investment only, it takes the maximum value: $k_1+k_2=\kappa L$ (with $k_1=\kappa\theta L$ and $k_2=\kappa(1-\theta)L$).

Using (4), the total demands (including transportation loss) for each variety are

$$m_{11}+\tau m_{12} = \frac{\mu w^{-\sigma}}{w^{1-\sigma}k_1+\phi k_2} Y_1 + \frac{\mu\phi w^{-\sigma}}{\phi w^{1-\sigma}k_1+k_2} Y_2,$$

$$m_{22}+\tau m_{21} = \frac{\mu}{\phi w^{1-\sigma}k_1+k_2} Y_2 + \frac{\mu\phi}{w^{1-\sigma}k_1+\phi k_2} Y_1, \tag{14}$$

where the national incomes are defined as

$$\begin{aligned} Y_1 &= r_1 k_{11} + \gamma r_2 k_{12} + w\theta L, \\ Y_2 &= \gamma r_1 k_{21} + r_2 k_{22} + (1-\theta)L. \end{aligned} \tag{15}$$

In the following sections we equate (10) with (14) to make a market clearing condition for a variety in each different case.

3 Equilibrium analysis with tradable Good A

This section describes an equilibrium analysis with tradable Good A; while next section shows the analysis with non-tradable Good A (because of sufficiently high transport cost). In this paper, an interior equilibrium means that there are positive number of firms in both countries simultaneously. Since we only consider the case of international capital movements from Country 2 to Country 1, k_1 and k_2 must be bounded as $k_1 \in (\theta\kappa L, [\theta+(1-\theta)\gamma]\kappa L)$ and $k_2 \in (0, (1-\theta)\kappa L)$; and $r_2=\gamma r_1$ holds in any interior equilibrium. In contrast, a corner equilibrium means full agglomeration in Country 1; thus, $k_1=[\theta+(1-\theta)\gamma]\kappa L$ and $k_2=0$ hold.

3.1 Interior equilibrium with tradable Good A

Assume that there are positive number of firms in both countries in an equilibrium; so that $r_2=\gamma r_1$, $k_1 \in (\theta\kappa L, [\theta+(1-\theta)\gamma]\kappa L)$ and $k_2 \in (0, (1-\theta)\kappa L)$ hold. If Country 1 imports Good A, then it holds that $p_1^A=w=t$ and the national incomes are

$$\begin{aligned} Y_1 &= (t+\kappa r_1)\theta L, \\ Y_2 &= (1+\gamma\kappa r_1)(1-\theta)L, \end{aligned} \tag{16}$$

and (10) makes

$$\frac{q_1}{q_2}=\frac{1}{\gamma t}. \tag{17}$$

Thus, the amounts of varieties produced in the other country differ from those in the domestic country unless $\gamma t=1$. The market clearing condition for a variety produced in Country 1, $q_1=m_{11}+\tau m_{12}$, can be expressed as

$$\sigma r_1=\frac{\mu\psi}{\psi k_1+\phi k_2}Y_1+\frac{\mu\psi\phi}{\psi\phi k_1+k_2}Y_2; \quad \psi\equiv t^{1-\sigma}, \tag{18}$$

where $\psi\leq 1$ represents trade freeness on Good A; and Y_1 and Y_2 are given by (16). Similarly, $q_2=m_{22}+\tau m_{21}$, can be expressed as

$$\sigma r_1=\frac{1}{\gamma}\left[\frac{\mu\phi}{\psi k_1+\phi k_2}Y_1+\frac{\mu}{\psi\phi k_1+k_2}Y_2\right]. \tag{19}$$

Substituting (19) into (18), we obtain the following important equation:

$$\frac{Y_1(\gamma\psi-\phi)}{\psi k_1+\phi k_2}=\frac{Y_2(1-\gamma\psi\phi)}{\psi\phi k_1+k_2} \tag{20}$$

where the left hand side (LHS) represents a profitability of a firm locates in Country 1, while the right hand side (RHS) represents that in Country 2. Each term in (20) has the following meaning respectively:

$$\underbrace{\frac{(price\ advantage)\times(market\ size)}{degree\ of\ competition}}_{\text{in Country 1}}=\underbrace{\frac{(price\ advantage)\times(market\ size)}{degree\ of\ competition}}_{\text{in Country 2}}, \tag{21}$$

where both the ***market size*** and ***price advantage*** have positive effects on the profitability, while the ***degree of competition*** has a negative effect on it. Whenever any interior equilibrium occurs, the profitability in each country must be equal.

From (20) we can obtain the following necessary condition for interior equilibrium:

$$\phi<\gamma\psi<\frac{1}{\phi}\quad\Leftrightarrow\quad\gamma^{\frac{1}{\sigma-1}}\frac{1}{\tau}<t<\gamma^{\frac{1}{\sigma-1}}\tau \tag{22}$$

Proof. As mentioned above the both sides of (20) must be equal for any interior equilibrium holds. Clearly, the RHS of (20) is positive, because $(1-\gamma\psi\phi)>0$; then it yields the second inequality in (22): $\gamma\psi<1/\phi$. Thus, the LHS of (20) must be positive as well and $(\gamma\psi-\phi)>0$; then it yields the first inequality in (22): $\phi<\gamma\psi$. **(Q.E.D.)**

Consider that Country 1 imports Good A from Country 2. Let I_M be the amount of imports as a function of r_1 and k_1; it can be obtained by

$I_M(r_1, k_1)$ = (Demand for Good A in Country 1) − (Production of Good A in Country 1)
= (Demand for Good A in Country 1) − (Population in Country 1)
+ (Labor employment in the manufacturing sector in Country 1)

$$=\frac{(1-\mu)(t+\kappa r_1)\theta L}{t}-\theta L+\frac{(\sigma-1)r_1k_1}{t}, \tag{23}$$

where (4) and (10) are used. Similarly, we can obtain the amount of exports, E_X , as a function of r_1 and k_2;

$E_X(r_1, k_2)$ = (Production of Good A in Country 1) − (Demand for Good A in Country 2)
= (Population in Country 2)

$-$ (Labor employment in the manufacturing sector in Country 2)
$-$ (Demand for Good A in Country 2)

$$= (1-\theta)L - (\sigma-1)\gamma r_1 k_2 - (1-\mu)(1+\gamma\kappa r_1)(1-\theta)L. \tag{24}$$

These two functions must satisfy the market clearing condition for tradable Good A, $E_X = t \times I_M$, with (13), the worldwide capital constraint. Then, we can solve the equation to get the equilibrium value of capital rent in Country 1:

$$r_1^* = \frac{\mu[1+\theta(t-1)]}{\kappa(\sigma-\mu)[\theta+(1-\theta)\gamma]} \tag{25}$$

which is increasing in t. (Each equilibrium value is marked with an asterisk.) Thus, any reduction in the transport cost of Good A decreases both the wage rate and capital rent in Country 1. Note that $E_X(=I_M)$ is monotonically decreasing in t; see (24). So the value of t satisfying $E_X = 0$ is uniquely determined; let denote the value by $\tilde{t}$ or $\tilde{w}$ (recall that $t = w$ in this case). We obtain the following lemma.

Lemma 1. *Consider an interior equilibrium with tradable Good A. In this case there is a threshold value $\tilde{t}$ of the transport cost of good A, so that, in the interior equilibrium, the larger country imports good A if $t \in [1, \tilde{t})$; otherwise good A is not traded.*

Unfortunately, we cannot solve for $\tilde{t}$ explicitly. However, the following lemma is helpful for understanding the endogenous determination of w in the next section with non-tradable Good A.

Lemma 2. *As long as Good A is tradable, it holds that $w = t \in [1, \tilde{t})$. However, if $t \geq \tilde{t}$, Lemma 1 shows that Good A is non-tradable and $w = \tilde{t}$ holds for any $t \in [\tilde{t}, \gamma^{\frac{1}{\sigma-1}}\tau)$.*

In other words, Lemma 2 states that there is no jump in value of w when the transport cost of t across the threshold value $\tilde{t}$.

Substituting (25) into (16), the equilibrium national incomes and the relative value of them are determined as the followings:

$$Y_1^* = \left[t + \frac{\mu[1+\theta(t-1)]}{(\sigma-\mu)[\theta+(1-\theta)\gamma]}\right]\theta L,$$

$$Y_2^* = \left[1 + \frac{\gamma\mu[1+\theta(t-1)]}{(\sigma-\mu)[\theta+(1-\theta)\gamma]}\right](1-\theta)L, \tag{26}$$

$$\frac{Y_1^*}{Y_2^*}=\frac{t(\sigma-\mu)[\theta+(1-\theta)\gamma]+\mu[1+\theta(t-1)]}{(\sigma-\mu)[\theta+(1-\theta)\gamma]+\gamma\mu[1+\theta(t-1)]}\cdot\frac{\theta}{1-\theta} > \frac{\theta}{1-\theta}. \tag{27}$$

Using these values, we can derive another necessary condition for interior equilibrium:

$$\phi < H < \frac{1}{\phi}, \quad \text{with } H \equiv \frac{(1-\gamma\psi\phi)Y_2^*}{(\gamma\psi-\phi)Y_1^*}. \tag{28}$$

Proof. Using (18) and (19), we can delete the second term in the RHS of (18) and (19); then we have

$$(1-\gamma\psi\phi)\sigma r_1^*=(1-\phi^2)\frac{\mu\psi Y_1^*}{\psi k_1+\phi k_2}. \tag{29}$$

We can also delete the first term in the RHS of (18) and (19) in the same way; then we have

$$(\gamma\psi-\phi)\sigma r_1^*=(1-\phi^2)\frac{\mu\psi Y_2^*}{\psi\phi k_1+k_2}. \tag{30}$$

By satisfying (29) and (30) simultaneously, k_1 and k_2 can be obtained as

$$k_1^*=\frac{-\mu}{\sigma r_1^*}\left[\frac{\phi Y_2^*}{\gamma\psi-\phi}-\frac{Y_1^*}{1-\gamma\psi\phi}\right], \tag{31}$$

$$k_2^*=\frac{\mu\psi}{\sigma r_1^*}\left[\frac{Y_2^*}{\gamma\psi-\phi}-\frac{\phi Y_1^*}{1-\gamma\psi\phi}\right]. \tag{32}$$

Since $k_1^* > 0$, the RHS of (31) must be positive, therefore, it must be

$$\phi(1-\gamma\psi\phi)Y_2^* < (\gamma\psi-\phi)Y_1^*. \tag{33}$$

Similarly, since $k_2^* > 0$, the RHS of (32) must be positive, therefore, it must be

$$\phi(\gamma\psi-\phi)Y_1^* < (1-\gamma\psi\phi)Y_2^*. \tag{34}$$

Combining (33) and (34) generates (28). **(Q.E.D.)**

The denominator and numerator of H are the mathematical product of the *price advantage* and the *market size* in each country. For simplicity, let call H as *relative investment factors* between two countries. (28) implies that the relative investment factors must be bounded for an interior equilibrium occurs.

We are mainly interested in whether the HME appears or not. Propositions 1, 2 and 3 in Takatsuka and Zeng (2012) state that the HME always appears in their model with perfect capital market integration. The reason behind their proposition also exists in our model with costly mobile capital (imperfect capital market integration); however, the share of firms' location is slightly modified to include the parameter γ. Following the literature, we states that the HME exists if the share of firms locating in the larger country, $k_1/(k_1+k_2)$, is greater than the population share of the larger country, $L_1/(L_1+L_2)=\theta$.

Using (30) and (32), we have the share of firms (invested effective capital) in Country 1:

$$\frac{k_1^*}{k_1^*+k_2^*}=\frac{\overbrace{\phi(1-\gamma\psi\phi)Y_2^*-(\gamma\psi-\phi)Y_1^*}^{\ominus}}{\underbrace{(\phi-\psi)(1-\gamma\psi\phi)Y_2^*}_{\ominus}+\underbrace{(1-\psi\phi)(\phi-\gamma\psi)Y_1^*}_{\ominus}}>0, \tag{35}$$

where the symbols ⊖ and ⊕ are used to show the terms are negative and positive, respectively. And it can be modified as

$$\frac{k_1^*}{k_1^*+k_2^*}-\theta=\frac{\overbrace{-(1-\theta+\psi\phi\theta)(\gamma\psi-\phi)Y_1^*}^{\ominus}+\overbrace{[(1-\theta)\phi+\psi\theta](1-\gamma\psi\phi)Y_2^*}^{\oplus}}{\underbrace{-(\psi-\phi)(1-\gamma\psi\phi)Y_2^*-(1-\psi\phi)(\gamma\psi-\phi)Y_1^*}_{\ominus}}, \tag{36}$$

to see whether if the HME occurs or not. Then we have the following results and Proposition 1:

$$\frac{k_1^*}{k_1^*+k_2^*}-\theta\begin{cases}>0 & \text{if} \quad H<\Lambda,\\ =0 & \text{if} \quad H=\Lambda,\\ <0 & \text{if} \quad H>\Lambda,\end{cases} \tag{37}$$

where

$$\Lambda\equiv\frac{1-\theta+\psi\phi\theta}{(1-\theta)\phi+\psi\theta}\in\left(\phi,\ \frac{1}{\phi}\right).$$

Proposition 1. *Consider an interior equilibrium with tradable Good A; and denote relative investment factors between two countries by H. In this case there is a threshold value,* Λ, *in H,* *whether if the HME occurs. If $H < \Lambda$, the HME occurs; otherwise the HME does not occur.*

Reconsidering Proposition 1 in two extreme cases: (i) $\phi = 1$, and (ii) $\phi = 0$; we obtain the following Corollary 1 and 2.

Corollary 1. *Assume that $\phi = 1$ ($\tau = 1$). In this case, the market for varieties is perfectly integrated, and the HME occurs.*

Proof. If $\phi = 1$, $\Lambda = 1$ and $H = -Y_2^*/Y_1^*$. Thus, $H < \Lambda$ holds and the HME occurs by Proposition 1. **(Q.E.D.)**

Corollary 2. *Assume that $\phi = 0$ ($\tau \to +\infty$). In this case, the market for varieties is divided, and firms locating in each country produce the amount as same as the domestic demands for their products. If $\gamma t > 1$, the HME occurs; otherwise the HME does not occur.*

Proof. If $\phi = 0$,

$$\Lambda = \frac{1-\theta}{\psi\theta},$$

$$H = \frac{1-\theta}{\psi\theta} \cdot \frac{(\sigma-\mu)[\theta+(1-\theta)\gamma]+\gamma\mu[1+\theta(t-1)]}{\gamma t(\sigma-\mu)[\theta+(1-\theta)\gamma]+\gamma\mu[1+\theta(t-1)]}.$$

Thus, these equations and Proposition 1 show that

$$\frac{k_1^*}{k_1^*+k_2^*} - \theta \begin{cases} > 0 & \text{if} \quad \gamma t > 1, \\ = 0 & \text{if} \quad \gamma t = 1, \\ < 0 & \text{if} \quad \gamma t < 1. \end{cases} \tag{38}$$

(Q.E.D.)

Next, the following lemma shows that the effects of a rise in γ on firms' location is generally indefinite.

Lemma 3. *The sign of a derivative, $\partial H/\partial\gamma$, is generally indefinite. However, if the transport cost of varieties, τ, is sufficiently small, the sign is positive: that is, $\partial H/\partial\gamma > 0$.*

Proof. See appendix.

Recall that a rise in γ means a reduction of the transport cost in foreign investment (or, in other words, an increase of the degree of capital market integration). Lemma 3 introduces the following proposition.

Proposition 2. *Consider an equilibrium, where the transport cost, τ, is sufficiently small so as to make $\partial H/\partial\gamma > 0$ hold. In this equilibrium, any increase of the degree of capital market integration contributes to improve the relative investment factors of the small country; and increases the share of firms in the smaller country (decreases that in the larger country).*

Note that the HME occurs as long as the transport cost, τ, is sufficiently small, as implied by Corollary 1. So Proposition 2 does not state that an increase of the degree of capital market integration generates a reversal of the HME. More integrated capital market with small value of τ redistributes capital (and firms) to the smaller country, but the HME still occurs. That is, the larger country still attracts more firms than its share of population.

As already mentioned in Lemmas 1 and 2, we cannot solve for $\tilde{t}$ explicitly, however, the value of $\hat{t}$ (which has same definition as $\tilde{t}$) in a case of a corner equilibrium can be solved explicitly. It is shown in the following subsection.

3.2 Corner equilibrium with tradable Good A

This subsection focus on a case of full agglomeration in Country 1: $k_1 = [\theta + (1-\theta)\gamma]\kappa L$, $k_2 = 0$ and $\gamma r_1 > r_2$ hold. Consider again that Country 1 imports Good A, then $p_1^{A} = w = t$ holds. The national incomes for this case are same as them in (16). Since $k_2 = 0$, (19) is not necessary as an equilibrium condition and (18) is transposed into

$$\sigma r_1 = \frac{\mu}{k_1}(Y_1 + Y_2). \tag{39}$$

This equation gives us the equilibrium value of capital rent in Country 1, which is same as one in (25).

In this case the amount of imports can be obtained by (24) with $k_1 = [\theta + (1-\theta)\gamma]\kappa L$ and r_1 in (25):

$$I_M(r_1,\ k_1) = \frac{(1-\mu)(t+\kappa r_1)\theta L}{t} - \theta L + \frac{(\sigma-1)r_1}{t}[\theta + (1-\theta)\gamma]\kappa L,$$

where I_M is monotonically decreasing in t. It can be solved for t satisfying that $I_M = 0$: denoted by $\hat{t}$

$$\hat{t} = \frac{\theta\,(\sigma - \mu) + \gamma\,(\sigma - 1)\,(1 - \theta)}{\theta\,\gamma\,(1 - \mu)}$$

We can show $\hat{t} > 1$ by a proof of contradiction.

Proof. If we assume that $\hat{t} < 1$, then it implies that $\theta\,(1 - \gamma)\,(\sigma - \mu) + \gamma\,(\sigma - 1) < 0$. However, this inequality cannot hold. Therefore, the assumption of $\hat{t} < 1$ is false, and $\hat{t} > 1$ is true. **(Q.E.D.)**

We obtain the following lemma.

Lemma 4. *Consider a corner equilibrium with tradable Good A. In this case there is a threshold value* $\hat{t}$ *of the transport cost of good A, so that, in the corner equilibrium, the larger country imports good A if* $t \in [1, \hat{t})$*; otherwise good A is not traded.*

4 Equilibrium analysis with non-tradable Good A

This section shows an equilibrium analysis with non-tradable Good A. Consider an interior equilibrium that $k_1, k_2 \in (0, \kappa L)$ and $r_2 = \gamma r_1$ hold. There is no international trade of Good A because shipping the good to the other country is very costly. In this case $p_1^A = w$ and $p_2^A = 1$ hold, and w is determined endogenously in equilibrium. The national incomes for this case are

$$\begin{aligned} Y_1 &= (w + \kappa r_1)\,\theta L, \\ Y_2 &= (1 + \gamma\,\kappa r_1)\,(1 - \theta)\,L, \end{aligned} \tag{40}$$

and (10) gives us the relative gap between q_1 and q_2 as

$$\frac{q_1}{q_2} = \frac{1}{\gamma w}, \tag{41}$$

therefore q_1 and q_2 do not equalize unless $\gamma w = 1$. The market clearing condition for a variety produced in Country 1, $q_1 = m_{11} + \tau m_{12}$, can be expressed as

$$\sigma r_1 = \frac{\mu\omega}{\omega k_1 + \phi k_2} Y_1 + \frac{\mu\omega\phi}{\omega\phi k_1 + k_2} Y_2; \quad \omega \equiv w^{1-\sigma}. \tag{42}$$

Similarly, the market clearing condition for a variety produced in Country 2, $q_2 = m_{22} + \tau m_{21}$, can be expressed as

$$\sigma r_1 = \frac{1}{\gamma}\left[\frac{\mu\phi}{\omega k_1 + \phi k_2} Y_1 + \frac{\mu}{\omega\phi k_1 + k_2} Y_2\right]. \tag{43}$$

Substituting (42) into (43), we obtain

$$\frac{Y_1(\gamma\omega - \phi)}{\omega k_1 + \phi k_2} = \frac{Y_2(1 - \gamma\omega\phi)}{\omega\phi k_1 + k_2} \tag{44}$$

where the LHS represents profitability of a firm locates in Country 1, while the RHS represents that in Country 2. From this equation we can obtain the following *necessary condition for interior equilibrium*:

$$\phi < \gamma\omega < \frac{1}{\phi} \quad \Leftrightarrow \quad \gamma^{\frac{1}{\sigma-1}}\frac{1}{\tau} < w < \gamma^{\frac{1}{\sigma-1}}\tau \tag{45}$$

Combining (44) with (13) gives us the equilibrium value of capital rent in Country 1:

$$r_1 = \frac{\mu[1 + \theta(w-1)]}{\kappa(\sigma - \mu)[\theta + (1-\theta)\gamma]}. \tag{46}$$

which depends positively on the value of w.

Next, we examine the labor constraint in each country. The labor employment in agricultural sector is determined from the domestic demand for Good A: $A_1 = (1-\mu)Y_1/w$ and $A_2 = (1-\mu)Y_2$. Subtracting each of them from the domestic labor endowment, θL and $(1-\theta)L$, we have the amount of labor supply for the manufacturing sector in each country: the LHS of (47) and (48) are the labor supply, respectively. Using (10), we obtain the amount of labor demand from the manufacturing sector: the RHS of (47) and (48). The market clearing condition for labor in each country can be shown as

$$\theta L\ [\mu w - (1-\mu)\kappa r_1] = (\sigma - 1) r_1 k_1 \quad \text{(for Country 1)}, \tag{47}$$

$$(1-\theta)L\ [\mu - (1-\mu)\gamma\kappa r_1] = (\sigma - 1) r_2 k_2 \quad \text{(for Country 2)}. \tag{48}$$

(47) gives us the number of firms in Country 1 as a function of r_1:

$$k_1 = \frac{\mu w - (1-\mu)\kappa \mathrm{r}_1}{(\sigma - 1) r_1}\theta L, \tag{49}$$

where w is increasing in r_1, as mentioned above. Similarly, (48) gives us the number of firms in Country 2 as a function of r_1:

$$k_2 = \frac{\mu - (1-\mu)\gamma\kappa r_1}{(\sigma-1)\gamma r_1}(1-\theta)L, \tag{50}$$

with

$$\frac{dk_2}{dr_1} = \frac{-(\sigma-1)\gamma\mu}{[(\sigma-1)\gamma r_1]^2}(1-\theta)L < 0.$$

Therefore,

$$\frac{dk_1}{dr_1} > 0,$$

since k_1 and k_2 cannot move to same direction simultaneously, shown by (13).

We are mainly interested in whether the HME appears or not. By using (46), (49) and (50), we obtain

$$\frac{k_1}{k_1+k_2} - \theta = \frac{\theta(1-\theta)(\sigma-\mu)[\theta+(1-\theta)\gamma](\gamma w-1)}{(\sigma-\mu)[\theta+(1-\theta)\gamma][1+\theta(\gamma w-1)] - \gamma(1-\mu)[1+\theta(w-1)]}. \tag{51}$$

Clearly, it holds that

$$\frac{k_1}{k_1+k_2} - \theta \begin{cases} > 0 & \text{if} \quad \gamma w > 1, \\ = 0 & \text{if} \quad \gamma w = 1, \\ < 0 & \text{if} \quad 0 < \gamma w < 1. \end{cases} \tag{52}$$

We obtain the following proposition.

Proposition 3. *Consider an interior equilibrium with non-tradable Good A. In this case there is a threshold value, $1/\gamma$, in w, whether if the HME occurs. If $w > 1/\gamma$, the HME occurs; otherwise the HME does not occur.*

Takatsuka and Zeng (2012) examine the case of $\gamma = 1$ only, and show that equilibrium value of w is larger than one in the case of non-tradable Good A. So they conclude that the HME always appears in that case (see the proposition 3 on page 1072 of their paper). Contrary to their paper, our model show that even if $w > 1$ always holds, γw can be smaller than one whenever we assume a sufficiently small value for γ. Thus, the HME does not necessarily appear in our model with costly mobile capital. The HME disappears if the equilibrium wage differential, w, is discounted largely by a sufficiently small value of γ.

5 Conclusion

In the literature on the HME, Helpman and Krugman (1985) shows the appearance of the HME formally by constructing a two-sector model. Then, many subsequent researches, such as Davis (1998) and Yu (2005), examine the conditions for the appearance of the HME in extended models of Helpman and Krugman (1985). Among all, Takatsuka and Zeng (2012) shows that the availability of mobile capital is crucial for the HME to appear, by using a footloose capital model of Martin and Rogers (1995). As predicted by Takatsuka and Zeng (2012), our model with costly foreign investment shows that an existence of large transport cost in capital movement hinders the appearance of the HME in an interior equilibrium with non-tradable Good A (see our Proposition 3); while the effect of a reduction in the cost (i.e., an increase in γ) on the appearance of the HME in an interior equilibrium with tradable Good A is generally indefinite (see our Proposition 1 and Lemma 3). In short, the HME does not always occur in our model with costly foreign investment.

Why is the effect of a reduction in the cost (i.e., an increase in γ) on the appearance of the HME unclear? Because it affects (i) the *relative price advantage*, and (ii) the *relative market size* between two countries in opposite directions. As shown in Appendix, an increase in γ always decreases the *relative price advantage* of the smaller country, while it always increases the *relative market size* of the smaller country. Our Proposition 2 shows that when the transport cost, τ, is sufficiently small, the latter effect dominates the former one, thus, it holds that $\partial H/\partial\gamma > 0$. In that case, a reduction of the transport cost in foreign investment (or, in other words, an increase of the degree of capital market integration) contributes to improve the relative investment factors of the small country; and increases the share of firms in the smaller country (decreases that in the larger country).

Our Proposition 2 is important when we think about market-opening policy of developing countries, since we often discuss that the sequence of market-opening is crucial for successful industrialization of developing countries. The proposition implies that further market-opening in capital market may contribute to attract firms (and capital) to less-developed (less-wealthier) countries as long as the market for industrial goods is already integrated sufficiently.

Appendix

To prove Lemma 3, we calculate the derivative, $\partial H/\partial\gamma$. Define

$$f(\gamma) \equiv \frac{1-\gamma\psi\phi}{\gamma\psi-\phi}, \quad g(\gamma) \equiv \frac{Y_2^*}{Y_1^*},$$

where we omit all asterisks for simplicity. Then, we have

$$f' \equiv \frac{df}{d\gamma} = \frac{\psi(\phi^2-1)}{(\gamma\psi-\phi)^2} < 0,$$

and

$$\begin{aligned} g' &\equiv \frac{dg}{d\gamma} \\ &= \frac{Y_1 Y_2' - Y_1' Y_2}{(Y_1)^2} \\ &= \frac{\mu(1-\theta)\theta[1+\theta(t-1)]\{\mu[1+\theta(t-1)]+(\sigma-\mu)[(1-\theta)(1-\gamma)+[\theta+(1-\theta)\gamma]t]\}}{\{t(\sigma-\mu)\theta[\theta+(1-\theta)\gamma]+\mu\theta[1+\theta(t-1)]\}^2} \\ &> 0, \end{aligned}$$

with

$$Y'_2 \equiv \frac{\partial Y_2}{\partial\gamma} > 0, \quad Y'_1 \equiv \frac{\partial Y_1}{\partial\gamma} < 0.$$

Therefore,

$$\begin{aligned} \frac{\partial H}{\partial\gamma} &= f'g + fg' \\ &= \frac{\psi\overbrace{(\phi^2-1)}^{\ominus} Y_1 Y_2 + \overbrace{(\gamma\psi-\phi)}^{\oplus}(1-\gamma\psi\phi)(Y_1 Y_2' - Y_1' Y_2)}{(\gamma\psi-\phi)^2(Y_1)^2}, \end{aligned}$$

where the sign of $\partial H/\partial\gamma$ is generally indefinite. However, since $\partial H/\partial\gamma > 0$ when $\phi = 1$, so we can suppose that $\partial H/\partial\gamma > 0$ holds as long as the value of ϕ is sufficiently small. **(Q.E.D.)**

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